MATH 121A Prep: Bases

1. Show that $\vec{e_1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\vec{e_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ form a basis for $\mathbb{R}^2$. Generalize this to a basis for $\mathbb{R}^n$. Conclude that $\mathbb{R}^n$ has dimension n. [Note: This is called the standard basis for $\mathbb{R}^n$.]

2. Prove that the vectors $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$, and $\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$ do not span $\mathbb{R}^3$.

3. Show that two vectors are linearly dependent if and only if one is a scalar multiple of the other.